LESSON

86 Number Families

WARM-UP

Facts Practice: Scientific Notation (Test S)

Mental Math:

a. $(-18) + (-40)$

b. $(3 \times 10^{-3})(3 \times 10^{-3})$

c. $7x + 4 = 60$

d. Estimate 15% of \$17.90.

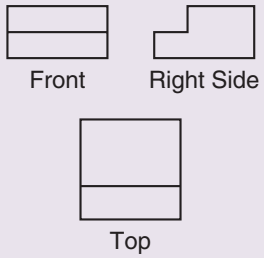
e. 0.2 L to mL

f. \$30 is $\frac{1}{3}$ of m.

g. What is the total cost of a \$200 item plus 7% sales tax?

Problem Solving:

Here are the front, top, and side views of an object. Draw a three-dimensional view of the object from the perspective of the upper right front.



NEW CONCEPT

In mathematics we give special names to certain sets of numbers. Some of these sets are the counting numbers, the whole numbers, the integers, and the rational numbers. In this lesson we will review each of these **number families** and discuss how they are related.

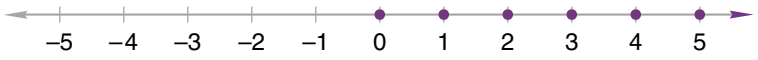
- **The Counting Numbers.** Counting numbers are the numbers we say when we count. The first counting number is 1, the next is 2, then 3, and so on.

  Counting numbers: 1, 2, 3, 4, 5, …

- **The Whole Numbers.** The members of the whole-number family are the counting numbers as well as the number zero.

  Whole numbers: 0, 1, 2, 3, 4, 5, …

  If we use a dot to mark each of the whole numbers on the number line, the graph looks like this:



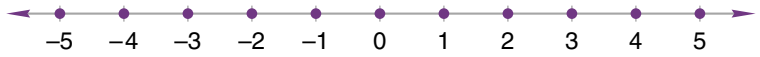
Notice that there are no dots to the left of zero. This is because no whole number is a negative number. Also

notice that there are no dots between consecutive whole numbers. Numbers between consecutive whole numbers are not "whole." The arrowhead on the right end of the number line indicates that the whole numbers increase without end.

- **The Integers.** The integer family includes all the whole numbers. It also includes the opposites (negatives) of the positive whole numbers. The list of integers goes on and on in both directions as indicated by the ellipses below.

  Integers: …, −4, −3, −2, −1, 0, 1, 2, 3, 4, …

  A graph of the integers looks like this:



  The arrowheads on both ends of the number line indicate that the set of integers continues without end in both directions. Notice that integers do not include such numbers as $\frac{1}{2}$, $\frac{5}{3}$, and other fractions.

- **The Rational Numbers.** The family of **rational numbers** includes all numbers that can be written as a *ratio* (fraction) of two integers. Here are some examples of rational numbers:

$$\frac{1}{2} \quad \frac{5}{3} \quad \frac{-3}{2} \quad \frac{-4}{1} \quad \frac{0}{2} \quad \frac{3}{1}$$

  Notice that the family of rational numbers includes all the integers, because every integer can be written as a fraction whose denominator is the number 1. For example, we can write −4 as a fraction by writing

$$\frac{-4}{1}$$

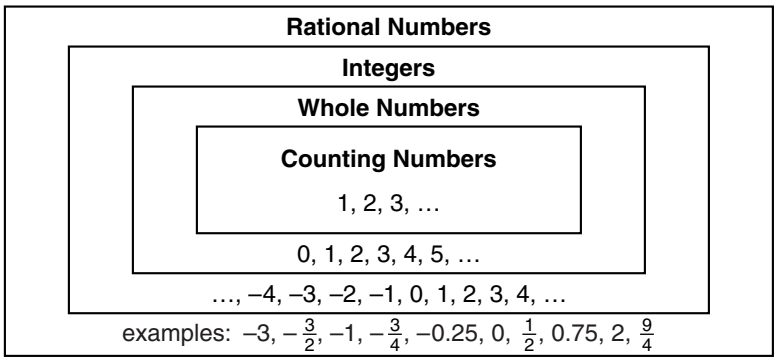
  The set of rational numbers also includes all the positive and negative mixed numbers, because these numbers can be written as fractions. For example, we can write $4\frac{1}{5}$ as

$$\frac{21}{5}$$

  Sometimes rational numbers are written in decimal form, in which case the decimal number will either terminate or repeat.

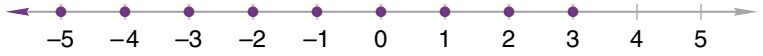
$$\frac{1}{8} = 0.125 \qquad \frac{5}{6} = 0.8333\ldots = 0.8\overline{3}$$

The diagram below may be helpful in visualizing the relationships between these families of numbers. The diagram shows that the set of rational numbers includes all the other number families described in this lesson.



Example 1 Graph the integers that are less than 4.

Solution We draw a number line and mark a dot at every integer that is less than 4. Since the set of integers includes whole numbers, we mark dots at 3, 2, 1, and 0. Since the integers also include the opposites of the positive whole numbers, we continue marking dots at –1, –2, –3, and so on. We then mark an arrowhead on the negative end of the line to indicate that the graph of integers that are less than 4 continues without end.



Example 2 Answer true or false:

(a) All whole numbers are integers.

(b) All rational numbers are integers.

Solution (a) **True.** Every whole number is included in the family of integers.

(b) **False.** Although every integer is a rational number, it is not true that every rational number is an integer. Rational numbers such as $\frac{1}{2}$ and $\frac{5}{3}$ are not integers.

LESSON PRACTICE

Practice set

a. Graph the integers that are greater than –4.

b. Graph the whole numbers that are less than 4.

Answer true or false:

c. Every integer is a whole number.

d. Every integer is a rational number.

MIXED PRACTICE

Problem set

1. (46) Heavenly Scent was priced at $28.50 for 3 ounces, while Eau de Rue cost only $4.96 for 8 ounces. Heavenly Scent cost how much more per ounce than Eau de Rue?

2. (65) Use a ratio box to solve this problem. The ratio of rookies to veterans in the camp was 2 to 7. Altogether there were 252 rookies and veterans in the camp. How many of them were rookies?

3. (Inv. 4) The seven linemen weighed 197 lb, 213 lb, 246 lb, 205 lb, 238 lb, 213 lb, and 207 lb. Find the (a) mode, (b) median, (c) mean, and (d) range of this group of measures.

4. (50) Use a unit multiplier to convert 12 bushels to pecks (1 bushel = 4 pecks).

5. (46) The Martins drove the car from 7 a.m. to 4 p.m. and traveled 468 miles. Their average speed was how many miles per hour?

6. (86) On a number line, graph the integers that are less than or equal to 3.

7. (72) Use a ratio box to solve this problem. Nine is to 6 as what number is to 30?

8. (22) Nine tenths of the company's 1800 employees attended the company picnic.

(a) How many of the company's employees attended the company picnic?

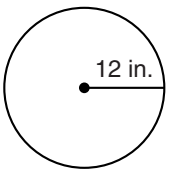
(b) What percent of the company's employees did not attend the company picnic?

9. (52) Evaluate: $\sqrt{b^2 - 4ac}$ if $a = 1$, $b = 5$, and $c = 4$

10. (79) Compare: $a^2 \bigcirc a$ if a is positive

11. (66, 82) (a) Find the circumference of the circle shown.

(b) Find the area of the circle.

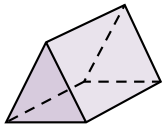


Leave π as π.

12. (47, 57) Find each missing exponent:

(a) $10^8 \cdot 10^{-3} = 10^{\square}$ (b) $10^5 \div 10^8 = 10^{\square}$

13. *(67)* The figure shown is a triangular prism. Copy the figure on your paper, and find the number of its (a) faces, (b) edges, and (c) vertices.



14. *(48)* Complete the table.

FRACTION	DECIMAL	PERCENT
(a)	0.9	(b)
$\frac{11}{12}$	(c)	(d)

15. *(17)* Obi is facing north. If he turns 360° in a clockwise direction, what direction will he be facing?

Use ratio boxes to solve problems 16 and 17.

16. *(81)* The sale price of $24 was 60 percent of the regular price. What was the regular price?

17. *(81)* Forty-eight corn seeds sprouted. This was 75 percent of the seeds that were planted. How many of the planted seeds did not sprout?

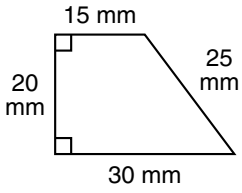
18. *(77)* Write an equation to solve this problem:

Thirty is what percent of 20?

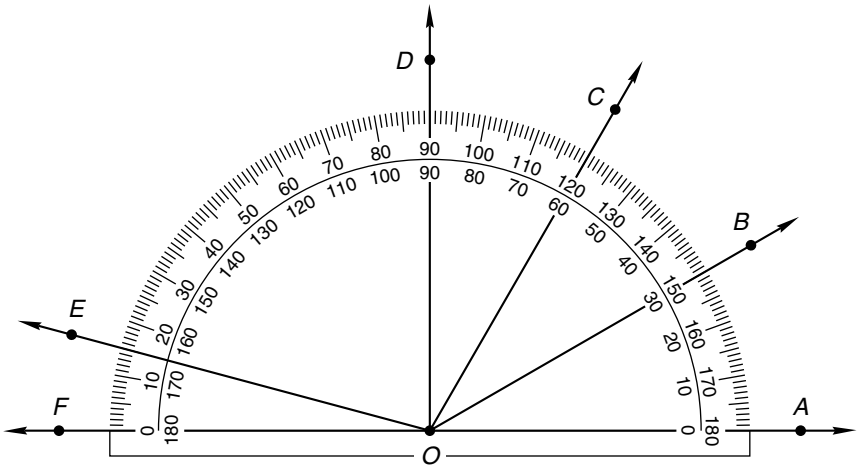
19. *(Inv. 6, 75)* (a) Classify the quadrilateral shown.

(b) Find its perimeter.

(c) Find its area.



20. *(17)* Find the measure of each angle.



(a) $\angle COF$ (b) $\angle AOE$ (c) $\angle BOE$

21. (85) Find the missing numbers in the table by using the function rule.

$y = 3x + 1$

x	y
4	☐
7	☐
0	☐

22. (83) Multiply. Write each product in scientific notation.

(a) $(1.2 \times 10^5)(1.2 \times 10^{-8})$

(b) $(6 \times 10^{-3})(7 \times 10^{-4})$

For problems 23 and 24, solve and check the equation. Show each step.

23. (Inv. 7) $56 = \frac{7}{8}w$

24. (Inv. 7) $4.8 + c = 7.34$

Simplify:

25. (52) $\sqrt{10^2 - 6^2} - \sqrt{10^2 - 8^2}$

26. (49)
$$\begin{array}{r} 5 \text{ lb } 9 \text{ oz} \\ +\ 4 \text{ lb } 7 \text{ oz} \\ \hline \end{array}$$

27. (43, 45) $1.4 \div 3\frac{1}{2} \times 10^3$ (decimal answer)

28. (85) (a) $(-4)(-5) - (-4)(+3)$ (b) $(-2)[(-3) - (-4)(+5)]$

29. (84) Collect like terms: $x^2 + 3xy + 2x^2 - xy$

30. (21) The factorization of $6x^2y$ is $2 \cdot 3 \cdot x \cdot x \cdot y$. Write the factorization of $9xy^2$.